

Run 5115 Col 5 Field 150 Coma cluster (Abell 1656) NGC 4874 z = 0.023 Run 3225 Col 4 Field 238 Abell 2199 z = 0.030

Hercules cluster (Abell 2151) z = 0.0366

III. THE VIRIAL THEOREM APPLIED TO CLUSTERS OF NEBULAE

If the total masses of clusters of nebulae were known, the average masses of cluster nebulae could immediately be determined from counts of nebulae in these clusters, provided internebular material is of the same density inside and outside of clusters.

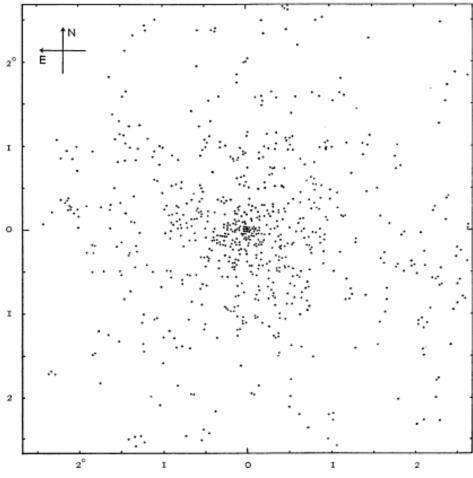


Fig. 3.—The Coma cluster of nebulae

As a first approximation, it is probably legitimate to assume that clusters of nebulae such as the Coma cluster (see Fig. 3) are mechanically stationary systems. With this assumption, the virial theorem of classical mechanics gives the total mass of a cluster in terms of the average square of the velocities of the individual nebulae which constitute this cluster.⁵ But even if we drop the assumption that clusclusters are gravitating systems in equilibrium - galaxies move on orbits that depend on the total amount of matter

the mass of the cluster can be determined by measuring the cluster size and the velocities of the galaxies



Moon: distance = $60.3 R_E$, period = 655 hours

geostationary satellite: $= 6.62 R_E$, period = 24 hours

Shuttle orbit: distance = $1.05 R_E$, period = 1.5 hours

Kepler's 3rd Law says that $R^3 / P^2 = constant$

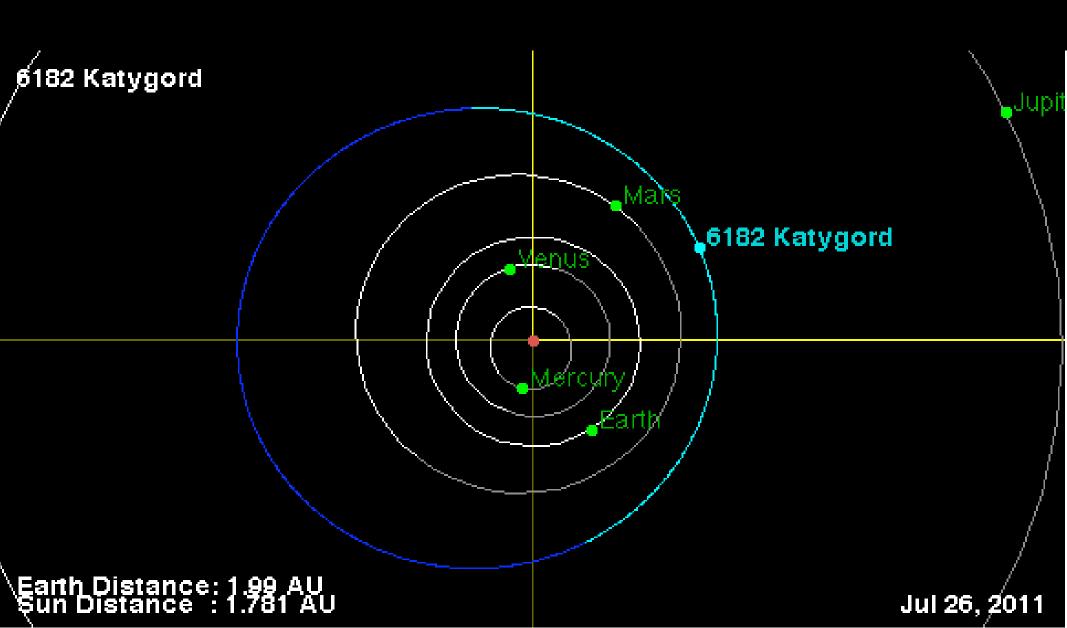
$$60.3^3 / 655^2 = 0.51$$

 $6.62^3 / 24^2 = 0.50$
 $1.05^3 / 1.5^2 = 0.51$

Jupiter and its four largest satellites

light takes 6.3 seconds to travel from Jupiter to Callisto

light takes 500 seconds (8.3 minutes) to travel from Sun to Earth



globular cluster Messier 3: 200 light-years in diameter



M 87, member of Virgo cluster of galaxies: 60,000 light-years in diameter



planet distance from Sun (AU)

period of orbit (yr)

a

P

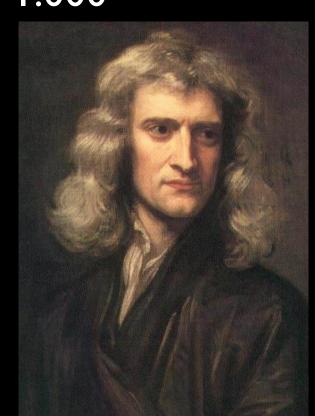
Mercury	0.387
Venus	0.723
Earth	1.000

0.241 0.615 1.000

 $a^3 / P^2 = 1.000 = mass of the Sun$



"Kepler's Third Law"



finding the mass of a cluster of galaxies

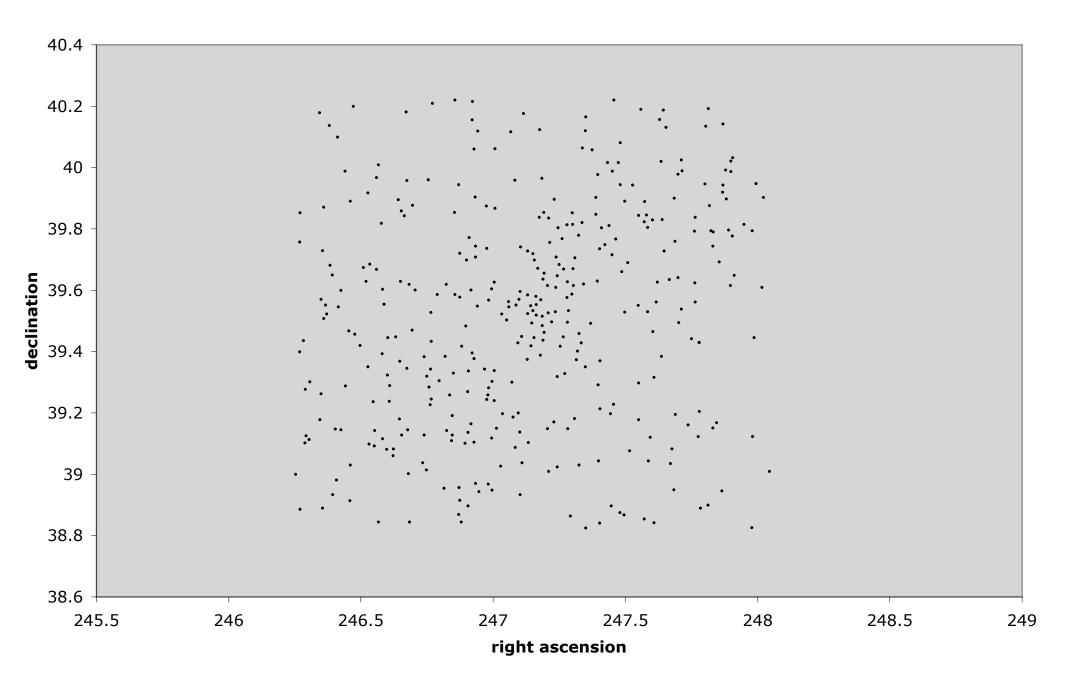
Kepler's 3rd Law: $M = a^3 / P^2$

a (semi-major axis) is measured in Astronomical Units (AU) P (period of the orbit) is measured in years M (mass of the cluster) is measured in solar masses

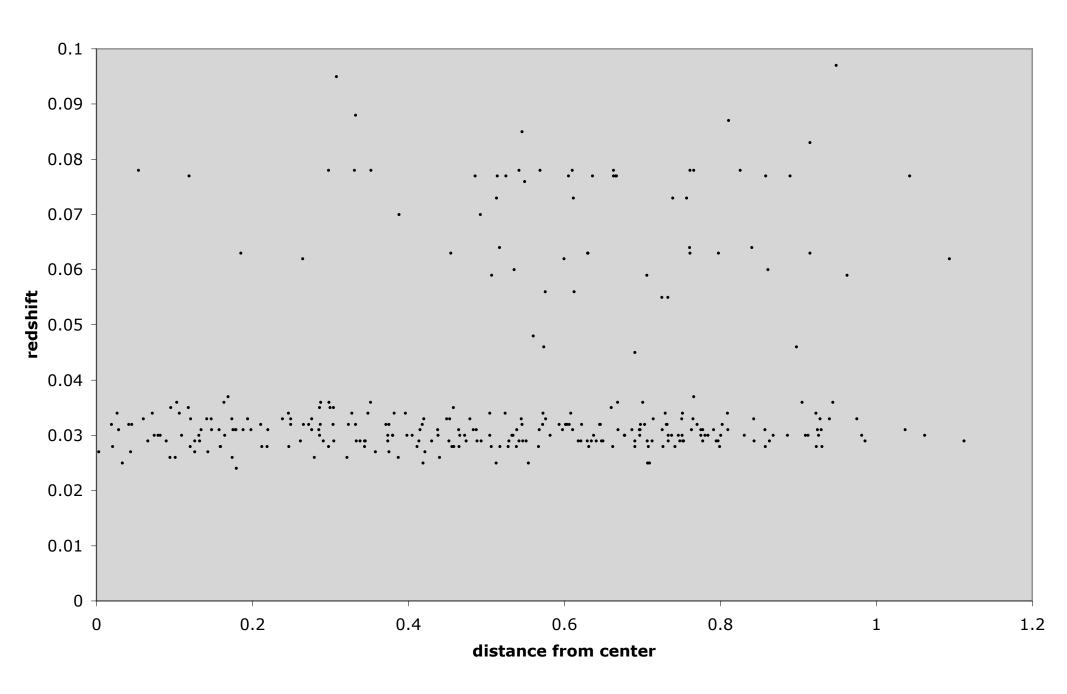
we can't measure P for galaxies in clusters (too long!), but we can measure a *velocity*.

velocity = distance / time = circumference / P = $2\pi a / P$

Abell 2199



Abell 2199



members of Abell 2199 have a mean redshift of z = 0.03distance = $c z / H_0 = 125 \text{ Mpc}$

the root-mean-square scatter around this mean is 0.0025

velocity dispersion = $c \times 0.0025 = 750 \text{ km/sec}$

velocities

low orbit around Earth 7.4 km/sec

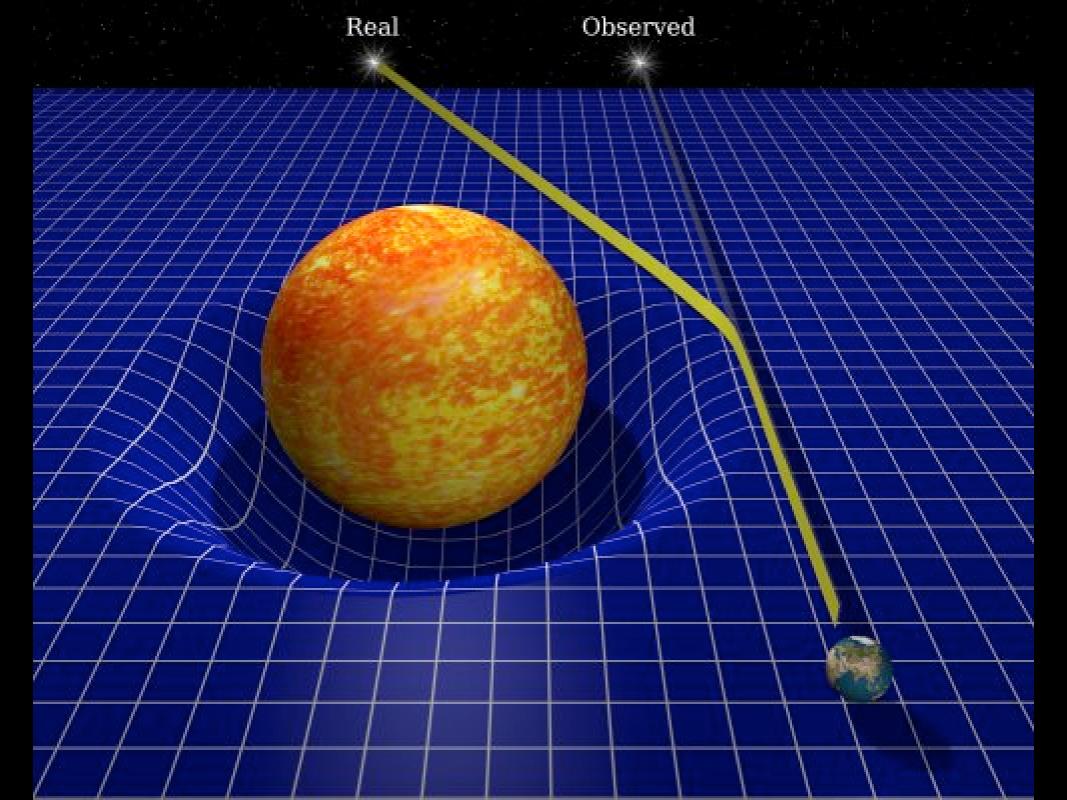
Earth escape velocity 11 km/sec

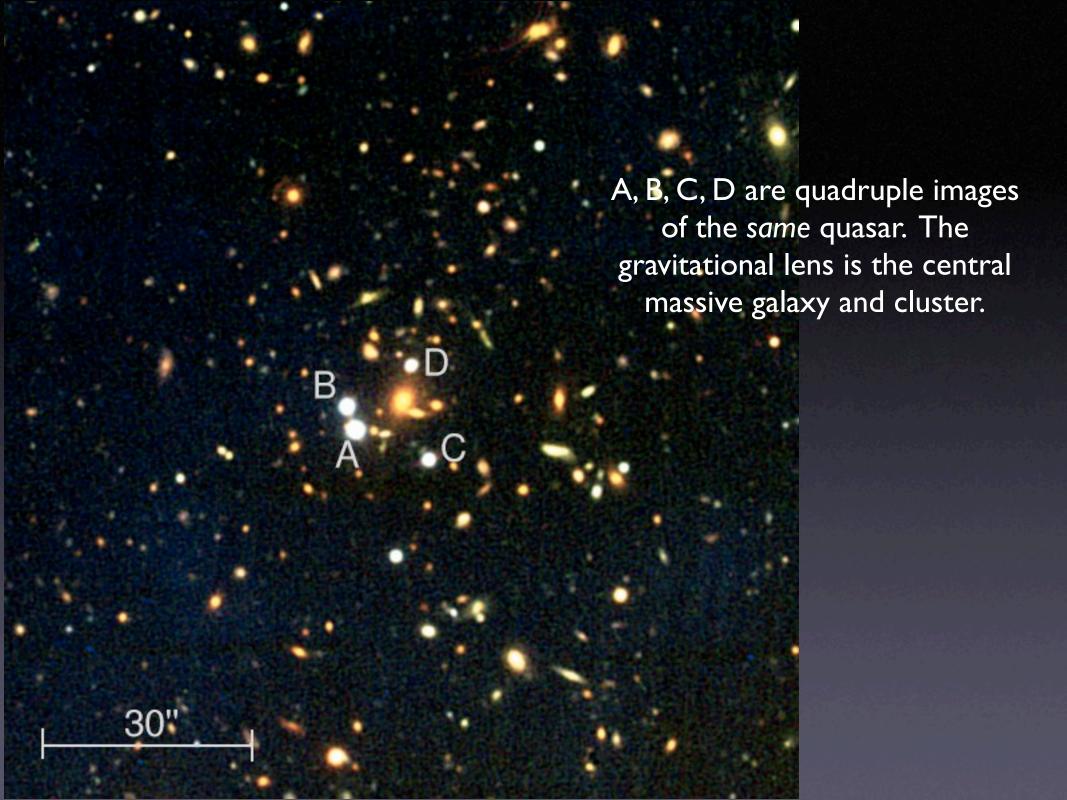
Mercury 48 km/sec

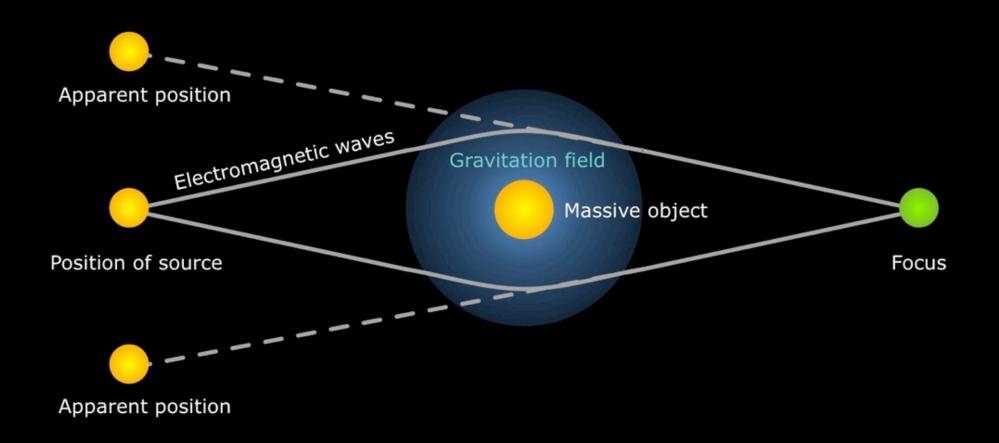
Earth 30 km/sec

Saturn 9.6 km/sec

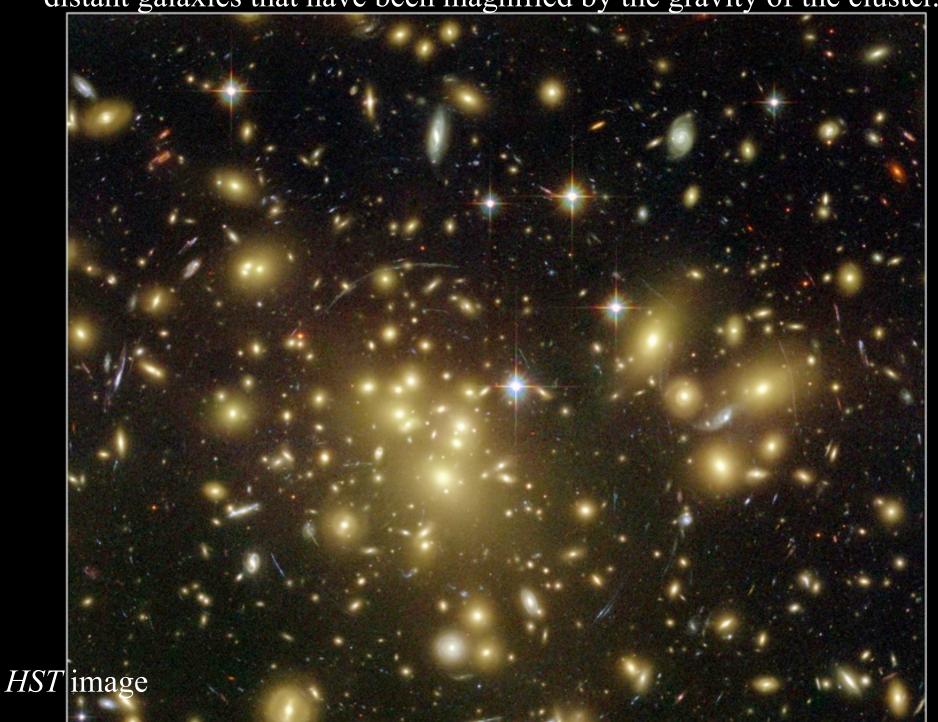
Sun around center of MW 220 km/sec light 300,000 km/sec







Rich cluster of galaxies (yellow). The small blue arcs are much more distant galaxies that have been magnified by the gravity of the cluster.













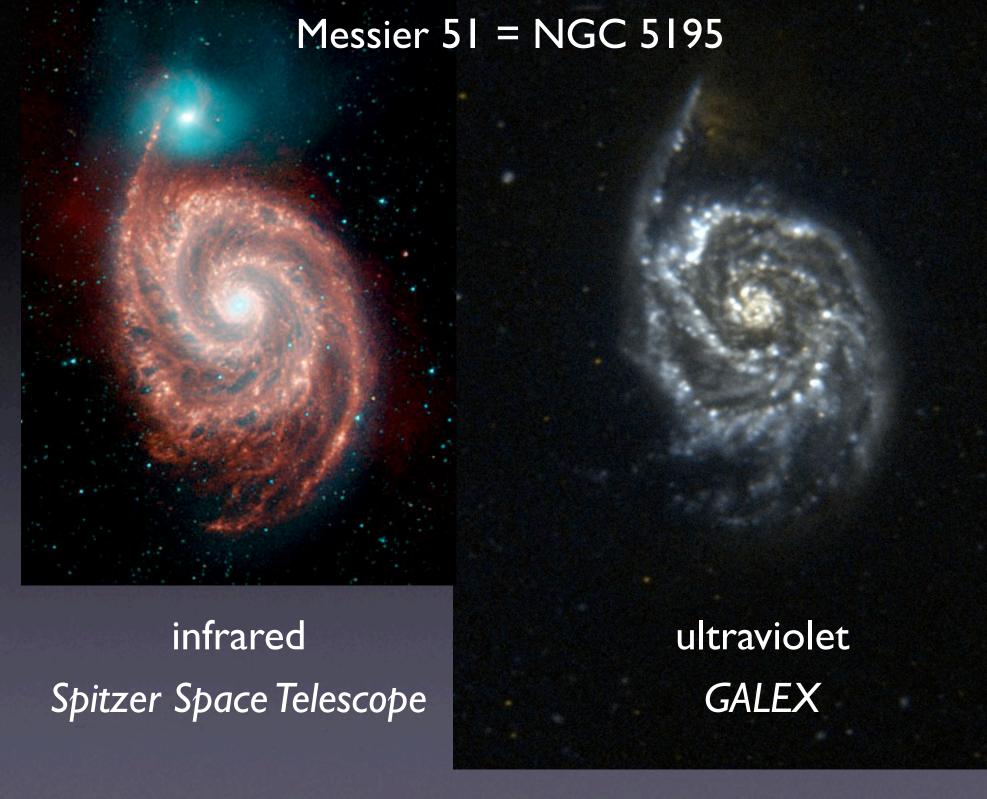


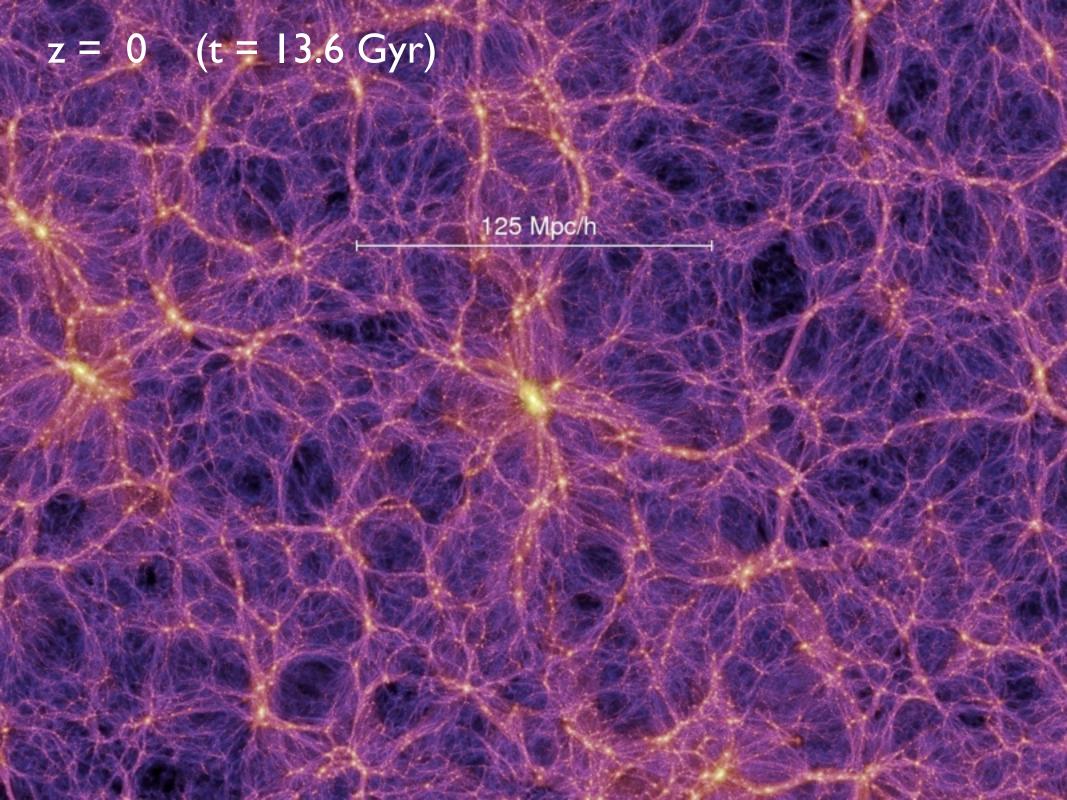


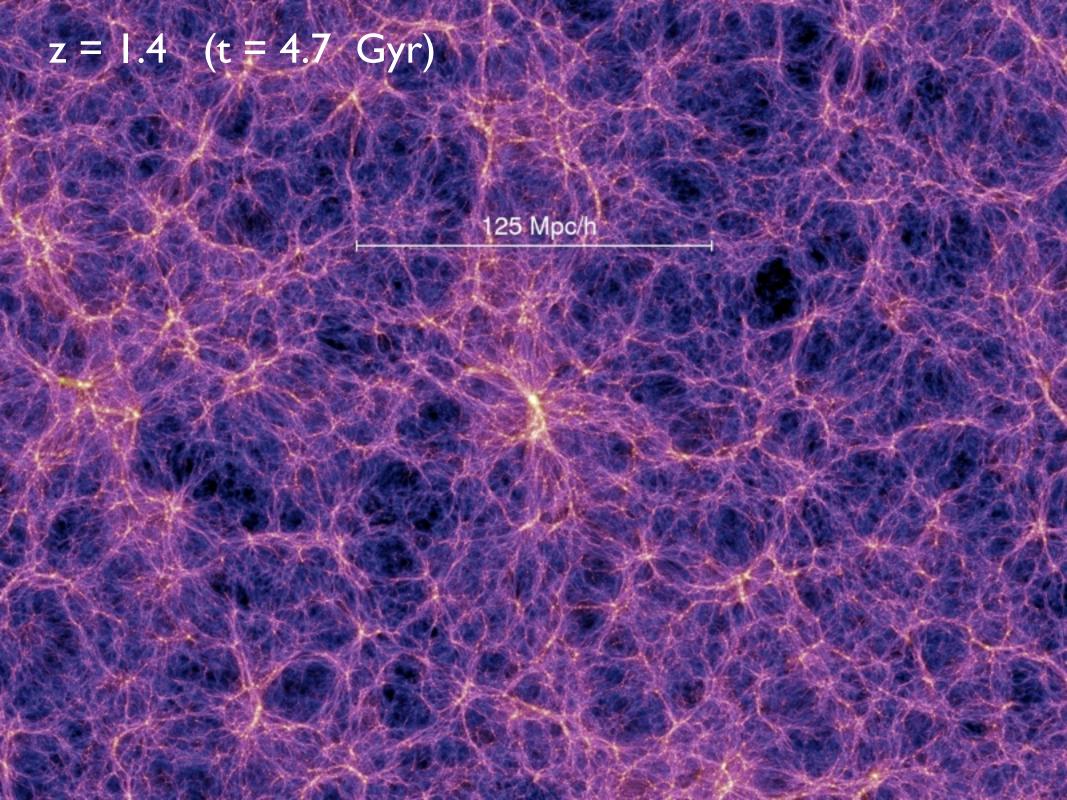


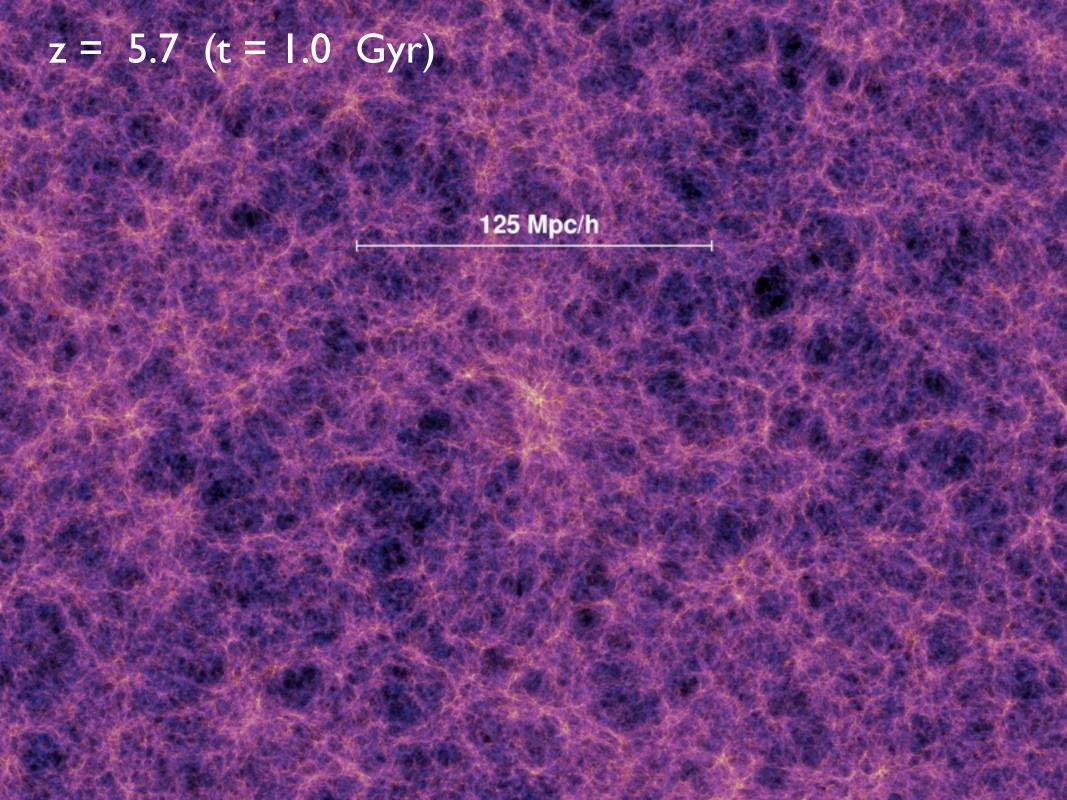












z = 18.3 (t = 0.21 Gyr)

125 Mpc/h

